Further Maths Summer Homework Task

Work through the examples in sections 1.2, 1.4 and 1.5, then complete exercises 1.2, 1.4 and 1.5. The solutions and hints are at the end of the document.

1.2 Algebraic Fractions

Many people have only a hazy idea of fractions. That needs improving if you want to go a long way with maths – you will need to be confident in handling fractions consisting of letters as well as numbers.

Remember, first, how to multiply a fraction by an integer. You multiply only the top *[what happens if you multiply both the top and the bottom of a fraction by the same thing?]*

Example 1 Multiply $\frac{4}{29}$ by 3.

Solution $4 \times 3 = 12$, so the answer is $\frac{12}{29}$.

Sometimes you can simplify the answer. If there is a common factor between the denominator (bottom) of the fraction and the number you are multiplying by, you can *divide* by that common factor.

Example 2 Multiply $\frac{7}{39}$ by 3.

Solution $39 \div 3 = 13$, so the answer is $\frac{7}{13}$.

You will remember that when you divide one fraction by another, you turn the one you are dividing by upside down, and multiply. If you are dividing by a whole number, you may need to write it as a fraction.

Example 3 Divide $\frac{7}{8}$ by 5.

Solution $\frac{7}{8} \div 5 = \frac{7}{8} \times \frac{1}{5}$, so the answer is $\frac{7}{40}$.

But if you can, you divide the top of the fraction only.

Example 4 Divide $\frac{20}{43}$ by 5.

Solution $\frac{20}{1} \times \frac{1}{5} = \frac{4}{1}$, so the answer is $\frac{4}{43}$. *Note* that you divide 20 by 5.

Do ***not*** multiply out 5×43; you'll only have to divide it again at the end!

Example 5 Multiply $\frac{3x}{7y}$ by 2.

Solution $3 \times 2 = 6x$, so the answer is $\frac{6x}{7y}$. (*Not* $\frac{6x}{14y}$!)

Example 6 Divide $\frac{3y^2}{4x}$ by y.

Solution $\frac{3y^2}{4x} \div y = \frac{3y^2}{4x} \times \frac{1}{y} = \frac{3y^2}{4xy} = \frac{3y}{4x}$, so the answer is $\frac{3y}{4x}$. [Don't forget to simplify.]

Example 7 Divide $\frac{PQR}{100}$ by T.

Solution $\frac{PQR}{100} \div T = \frac{PQR}{100} \times \frac{1}{T} = \frac{PQR}{100T}$.

Here it would be wrong to say just $\frac{\frac{PQR}{100}}{T}$, which is a mix (as well as a mess!)

Double fractions, or mixtures of fractions and decimals, are always wrong.

For instance, if you want to divide $\frac{xy}{z}$ by 2, you should not say $\frac{0.5xy}{z}$ but $\frac{xy}{2z}$.

This sort of thing is extremely important when it comes to rearranging formulae.

Example 8 Make r the subject of the equation $V = \frac{1}{2}\pi r^2 h$.

Solution Multiply by 2: $2V = \pi r^2 h$

Don't "divide by $\frac{1}{2}$".

Divide by π and h: $\frac{2V}{\pi h} = r^2$

Square root both sides: $r = \sqrt{\frac{2V}{\pi h}}$.

You should *not* write the answer as $\sqrt{\dfrac{V}{\frac{1}{2}\pi h}}$ or $\sqrt{\dfrac{2V}{\pi} \div h}$, as these are fractions of fractions.

Make sure, too, that you write the answer properly. If you write $\sqrt{}2V/\pi h$ it's not at all clear that the whole expression has to be square-rooted and you will lose marks.

If you do get a compound fraction (a fraction in which either the numerator or the denominator, or both, contain one or more fractions), you can always simplify it by multiplying all the terms, on both top and bottom, by any *inner denominators*.

Example 9 Simplify $\dfrac{\dfrac{1}{x-1}+1}{\dfrac{1}{x-1}-1}$.

Solution Multiply all four terms, on both top and bottom, by $(x - 1)$:

$$\frac{\dfrac{1}{x-1}+1}{\dfrac{1}{x-1}-1} = \frac{\dfrac{(x-1)}{x-1}+(x-1)}{\dfrac{(x-1)}{x-1}-(x-1)}$$

$$= \frac{1+(x-1)}{1-(x-1)}$$

$$= \frac{x}{2-x}$$

You will often want to combine two algebraic expressions, one of which is an algebraic fraction, into a single expression. You will no doubt remember how to add or subtract fractions, using a common denominator.

Example 10 Simplify $\dfrac{3}{x-1} - \dfrac{1}{x+1}$.

Solution Use a common denominator. [You must treat $(x - 1)$ and $(x + 1)$ as separate expressions with no common factor.]

$$\frac{3}{x-1} - \frac{1}{x+1} = \frac{3(x+1)-(x-1)}{(x-1)(x+1)}$$

$$= \frac{3x+3-x+1}{(x-1)(x+1)} = \frac{2x+4}{(x-1)(x+1)}.$$

Do use brackets, particularly on top – otherwise you are likely to forget the minus at the end of the numerator (in this example subtracting -1 gives +1).

Don't multiply out the brackets on the bottom. You will need to see if there is a factor which cancels out (although there isn't one in this case).

Example 11 Simplify $\frac{2}{3x-3}+\frac{5}{x^2-1}$.

Solution A common denominator may not be obvious, you should look to see if the denominator factorises first.

$$\frac{2}{3x-3}+\frac{5}{x^2-1}=\frac{2}{3(x-1)}+\frac{5}{(x+1)(x-1)}$$

$$=\frac{2(x+1)+5\times 3}{3(x-1)(x+1)}$$

$$=\frac{2x+2+15}{3(x-1)(x+1)}$$

$$=\frac{2x+17}{3(x-1)(x+1)}$$

$x-1$ is a common factor, so the common denominator is $3(x-1)(x+1)$.

If one of the terms is not a fraction already, the best plan is to make it one.

Example 12 Write $\frac{3}{x+1}+2$ as a single fraction.

Solution

$$\frac{3}{x+1}+2=\frac{3}{x+1}+\frac{2}{1}$$

$$=\frac{3+2(x+1)}{x+1}$$

$$=\frac{2x+5}{x+1}$$

This method often produces big simplifications when roots are involved.

Example 13 Write $\frac{x}{\sqrt{x-2}}+\sqrt{x-2}$ as a single fraction.

Solution

$$\frac{x}{\sqrt{x-2}}+\sqrt{x-2}=\frac{x}{\sqrt{x-2}}+\frac{\sqrt{x-2}}{1}$$

$$=\frac{x+(\sqrt{x-2})^2}{\sqrt{x-2}}$$

$$=\frac{x+(x-2)}{\sqrt{x-2}}$$

$$=\frac{2x-2}{\sqrt{x-2}}$$

It is also often useful to reverse this process – that is, to rewrite expressions such as $\frac{x}{x-2}$.

The problem with this expression is that x appears in more than one place and it is not very easy to manipulate such expressions (for example, in finding the inverse function, or sketching a curve). Here is a very useful trick.

Example 14 Write $\frac{x}{x-2}$ in the form $a+\frac{b}{x-2}$, where a and b are integers.

Solution

$$\frac{x}{x-2}=\frac{(x-2)+2}{x-2}$$

Write “the top” as “the bottom plus or minus a number”.

$$=\frac{x-2}{x-2}+\frac{2}{x-2}$$

$$=1+\frac{2}{x-2}$$

Example 15 Write the equation $\frac{1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$ without fractions.

(A and B are constants that remain in your answer.)

Solution Multiply both sides by the common denominator, here $(x-2)(x+1)$:

$$1 = \frac{A(x-2)(x+1)}{(x-2)} + \frac{B(x-2)(x+1)}{(x+1)}$$

Cancel out the common factors.

$$1 = A(x+1) + B(x-2)$$

This is an important technique in A Level.

Exercise 1.2

1 Work out the following. Answers may be left as improper fractions.

(a) $\frac{4}{7} \times 5$ (b) $\frac{5}{12} \times 3$ (c) $\frac{7}{9} \times 2$ (d) $\frac{4}{15} \times 3$

(e) $\frac{8}{11} \div 4$ (f) $\frac{8}{11} \div 3$ (g) $\frac{6}{7} \div 3$ (h) $\frac{6}{7} \div 5$

(i) $\frac{3x}{y} \times x$ (j) $\frac{3x}{y^2} \times y$ (k) $\frac{5x^3}{4y} \div x$ (l) $\frac{5x^2}{6y} \div y$

(m) $\frac{5x^3}{2y} \times 3x$ (n) $\frac{3y^4}{4x^2z} \times 2x$ (o) $\frac{6x^2y^3}{5z} \div 2xy$ (p) $\frac{5a^2}{6x^3z^2} \div 2y$

2 Make x the subject of the following formulae.

(a) $\frac{1}{2}A = \pi x^2$ (b) $V = \frac{4}{3}\pi x^3$ (c) $\frac{1}{2}(u+v) = tx$ (d) $W = \frac{2}{3}\pi x^2 h$

3 Simplify the following compound fractions.

(a) $\dfrac{\frac{1}{x}+1}{\frac{1}{x}+3}$ (b) $\dfrac{\frac{2}{x}+1}{\frac{3}{x}-1}$ (c) $\dfrac{\frac{1}{x+1}+2}{\frac{1}{x+1}+1}$

4 Write as single fractions.

(a) $\dfrac{2}{x-1}+\dfrac{1}{x+3}$ (b) $\dfrac{2}{x-3}-\dfrac{1}{x+2}$ (c) $\dfrac{1}{2x-1}-\dfrac{1}{3x+2}$ (d) $\dfrac{3}{x+2}+1$

(e) $2-\dfrac{1}{x-1}$ (f) $\dfrac{2x}{x+1}-3$ (g) $\dfrac{3}{4(2x-1)}-\dfrac{1}{4x^2-1}$

5 Write as single fractions.

(a) $\dfrac{x+1}{\sqrt{x}}+\sqrt{x}$ (b) $\dfrac{2x}{\sqrt{x+3}}+\sqrt{x+3}$ (c) $\dfrac{x}{\sqrt[3]{x-2}}+\sqrt[3]{(x-2)^2}$

6 Write the following in the form $1+\dfrac{a}{x+b}$.

(a) $\dfrac{x+1}{x-5}$ (b) $\dfrac{x+3}{x+1}$ (c) $\dfrac{x+2}{x+5}$ (d) $\dfrac{x-6}{x-2}$

7 Write the following equations without fractions. (A, B etc. are constants that remain in your answers.)

(a) $\dfrac{1}{(x-2)(x+1)}=\dfrac{A}{x-2}+\dfrac{B}{x+1}$

(b) $\dfrac{x+2}{(x+2)(x-3)}=\dfrac{A}{x+2}+\dfrac{B}{x-3}$

(c) $\dfrac{2}{(x+1)(x+2)(x-3)}=\dfrac{A}{x+1}+\dfrac{B}{x+2}+\dfrac{C}{x-3}$

(d) $\dfrac{1}{(x-2)^2(x+1)}=\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{x+1}$

(e) $\dfrac{1}{x^2(x+2)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+2}$

1.4 Cancelling

The word “cancel” is a very dangerous one. It means two different things, one safe enough and the other very likely to lead you astray.

You can cancel *like terms* when they are added or subtracted.

Example 1 Simplify $(x^2 - 3xy) + (3xy - y^2)$.

Solution $(x^2 - 3xy) + (3xy - y^2) = x^2 - \cancel{3xy} + \cancel{3xy} - y^2 = x^2 - y^2$.

The “$3xy$” terms have “cancelled out”. This is safe enough.

It is also usual to talk about “cancelling down a fraction”. Thus $\frac{10}{15} = \frac{2}{3}$. However, this tends to be very dangerous with anything other than the most straightforward numerical fractions. Consider, for instance, a fraction such as $\frac{x^2 + 2xy}{xy + 2y^2}$. If you try to “cancel” this, you're almost certain not to get the right answer, which is in fact $\frac{x}{y}$ (as we will see in Example 4, below).

Try instead to use the word “divide”. What happens when you “cancel down” $\frac{10}{15}$ is that you *divide top and bottom* by 5. If you can divide both the top and bottom of a fraction by the same thing, this is a correct thing to do and you will get a simplified answer.

Contrast these two examples: $\frac{4x + 8y}{4}$ and $\frac{4x \times 8y}{4}$.

In the first, you can divide both $4x$ and $8y$ by 4 and get $x + 2y$, which is the correct answer (though it is rather safer to start by factorising the top to get $4(x + 2y)$, after which it is obvious that you can divide top and bottom by 4.)

In the second example, you don't do the same thing. $4x \times 8y = 32xy$. This can be divided by 4 to get $8xy$, which is the correct answer. Apparently here only one of the two numbers, 4 and 8, has been divided by 4, whereas before both of them were. That is true, but it's not a very helpful way of thinking about it.

With problems like these, start by multiplying together any terms that you can (like the $4x$ and the $8y$ in the second example). Then, if you can, factorise the whole of the top and/or the bottom of a fraction before doing any “cancelling”. Then you will be able to see whether you can divide out any common factors.

Example 2 Simplify $\frac{4x+6y}{12x+6y}$.

Solution $\frac{4x+6y}{12x+6y} = \frac{2(2x+3y)}{6(2x+y)} = \frac{2x+3y}{3(2x+y)}$

The top factorises as $2(2x + 3y)$. The bottom factorises as $6(2x + y)$.

2 and 6 have a common factor of 2, which can be divided out to give 3.

But $(2x + 3y)$ and $(2x + y)$ have *no common factor* (neither 2 nor x divides into $3y$ or y, and neither 3 nor y divides into $2x$).

So you can't go any further, and the answer is $\frac{(2x+3y)}{3(2x+y)}$.

Example 3 Explain why you cannot cancel down $\frac{x^2+3y^3}{3x^2+1}$.

Solution There is nothing that divides all four terms (x^2, $3y^2$, $3x^2$ and 1), and neither the top nor the bottom can be factorised. So nothing can be done.

Example 4 Simplify $\frac{x^2+2xy}{xy+2y^2}$.

Solution Factorise the top as $x(x + 2y)$ and the bottom as $y(x + 2y)$:

$$\frac{x^2+2xy}{xy+2y^2} = \frac{x(x+2y)}{y(x+2y)}$$

Now it is clear that both the top and the bottom have a factor of $(x + 2y)$.

So this can be divided out to give the answer of $\frac{x}{y}$.

Don't "cancel down". Factorise if you can; divide all the top and all the bottom.

Taking out factors

I am sure you know that $7x^2 + 12x^3$ can be factorised as $x^2(7 + 12x)$.

You should be prepared to factorise an expression such as $7(x + 2)^2 + 12(x + 2)^3$ in the same way.

Example 5 Factorise $7(x+2)^2 + 12(x+2)^3$

Solution $7(x+2)^2 + 12(x+2)^3 = (x+2)^2(7 + 12(x+2))$

$$= (x+2)^2(12x+31).$$

The only differences between this and $7x^2 + 12x^3$ are that the common factor is $(x+2)^2$ and not x^2; and that the other factor, here $(7 + 12(x+2))$, can be simplified.

If you multiply out the brackets you will get a cubic and you will have great difficulty in factorising that. ***Don't multiply out brackets if you can help it!***

Expressions such as those in the next exercise, question 4 parts (c) and (d) and question 5 parts (e)–(h), occasionally arise in two standard techniques, the former in Further Mathematics (Mathematical Induction) and the latter in A2 Mathematics (the Product and Quotient Rules for differentiation). They may look a bit intimidating at this stage; feel free to omit them if you are worried by them.

Exercise 1.4

1 Simplify the following as far as possible.

(a) $5x + 3y + 7x - 3y$ (b) $3x^2 + 4xy + y^2 + x^2 - 4xy - y^2$.

(c) $\dfrac{4+6x}{2}$ (d) $\dfrac{4\times 6x}{2}$ (e) $\dfrac{3x+xy}{x}$

(f) $\dfrac{3x\times xy}{x}$ (g) $\dfrac{4x+10y}{8x+6y}$ (h) $\dfrac{3x-6y}{9x-3y}$

(i) $\dfrac{4x+9y}{2x+3y}$ (j) $\dfrac{4x+6y}{6x+9y}$ (k) $\dfrac{5xy+6y^2}{10x+12y}$

(l) $\dfrac{3x^2+4y^2}{6x^2-8y^2}$ (m) $\dfrac{x-3}{3-x}$ (n) $\dfrac{x^2-2xy-y^2}{y^2+2xy-x^2}$

2 Make x the subject of the following formulae.

(a) $\dfrac{ax}{b} = \dfrac{py}{qz}$ (b) $\dfrac{3\pi ax}{b} = \dfrac{4y^2}{qz}$

3 Simplify the following.

(a) $\dfrac{2\pi x}{ab} \div \frac{1}{3}\pi r^3$ (b) $\dfrac{2\pi h^2}{rb} \div \frac{4}{3}\pi h r^2$

4 Simplify into a single factorised expression.

(a) $(x-3)^2 + 5(x-3)^3$ (b) $4x(2x+1)^3 + 5(2x+1)^4$

(c) $\frac{1}{2}k(k+1) + (k+1)$ (d) $\frac{1}{6}k(k+1)(2k+1) + (k+1)^2$

5 Simplify as far as possible.

(a) $\dfrac{x^2+6x+8}{x^2-x-6}$ (b) $\dfrac{3x^2-2x-8}{x^2-4}$

(c) $\dfrac{(x+3)^2-2(x+3)}{x^2+2x-3}$ (d) $\dfrac{x(2x-1)^2-x^2(2x-1)}{(x-1)^2}$

(e) $\dfrac{\dfrac{x^2}{\sqrt{x^2+1}}-\sqrt{x^2+1}}{x^2}$ (f) $-\dfrac{\dfrac{x}{2\sqrt{1-x}}+\sqrt{1-x}}{x^2}$

(g) $\dfrac{\dfrac{\sqrt{x}}{2\sqrt{1+x}}-\dfrac{\sqrt{1+x}}{2\sqrt{x}}}{x}$ (h) $\dfrac{\sqrt[3]{1+x}-\dfrac{x}{3\sqrt[3]{(1+x)^2}}}{\sqrt[3]{1+x}}$

1.5 Simultaneous equations

I am sure that you will be very familiar with the standard methods of solving simultaneous equations (elimination and substitution). You will probably have met the method for solving simultaneous equations when one equation is linear and one is quadratic. Here you have no choice; you ***must*** use substitution.

Example 1 Solve the simultaneous equations $x + 3y = 6$

$$x^2 + y^2 = 10$$

Solution Make one letter the subject of the linear equation: $x = 6 - 3y$

Substitute into the quadratic equation $(6 - 3y)^2 + y^2 = 10$

Solve … $10y^2 - 36y + 26 = 0$

$$2(y - 1)(5y - 13) = 0$$

… to get two solutions: $y = 1 \text{ or } 2.6$

Substitute both back into the *linear* equation $x = 6 - 3y = 3 \text{ or } -1.8$

Write answers in pairs: $(x, y) = (3, 1) \text{ or } (-1.8, 2.6)$

- You can't just square root the quadratic equation. *[Why not?]*
- You could have substituted for y instead of x (though in this case that would have taken longer – try to avoid fractions if you can).
- It is very easy to make mistakes here. Take great care over accuracy.
- It is remarkably difficult to *set* questions of this sort in such a way that *both* pairs of answers are nice numbers. Don't worry if, as in this example, only *one* pair of answers are nice numbers.

Questions like this appear in many GCSE papers. They are often, however, rather simple (sometimes the quadratic equations are restricted to those of the form $x^2 + y^2 = a$) and it is important to practice less convenient examples.

Exercise 1.5

Solve the following simultaneous equations.

1 $x^2 + xy = 12$

$3x + y = 10$

2 $x^2 - 4x + y^2 = 21$

$y = 3x - 21$

3 $x^2 + xy + y^2 = 1$

$x + 2y = -1$

4 $x^2 - 2xy + y^2 = 1$

$y = 2x$

5 $c^2 + d^2 = 5$

$3c + 4d = 2$

6 $x + 2y = 15$

$xy = 28$

7 $2x^2 + 3xy + y^2 = 6$

$3x + 4y = 1$

8 $2x^2 + 4xy + 6y^2 = 4$

$2x + 3y = 1$

9 $4x^2 + y^2 = 17$

$2x + y = 5$

10 $2x^2 - 3xy + y^2 = 0$

$x + y = 9$

11 $x^2 + 3xy + 5y^2 = 15$

$x - y = 1$

12 $xy + x^2 + y^2 = 7$

$x - 3y = 5$

13 $x^2 + 3xy + 5y^2 = 5$

$x - 2y = 1$

14 $4x^2 - 4xy - 3y^2 = 20$

$2x - 3y = 10$

15 $x^2 - y^2 = 11$

$x - y = 11$

16 $\frac{12}{x} + \frac{1}{y} = 3$

$x + y = 7$

Further reading

There are not many books designed for the sort of transition that this booklet represents, but an outstanding exception is:

Fyfe, M. T., Jobbings, A. and Kilday, K. (2007) *Progress to Higher Mathematics*, Arbelos.

ISBN 9780955547706.

Alternatively, also published by Arbelos is an expansion of the same book which is more specifically aimed at the transition to A Level. You may not need quite so much as this:

Fyfe, M. T., and Kilday, K. (2011) *Progress to Advanced Mathematics*, Arbelos.

ISBN 9780955547737.

Answers, hints and comments

Exercise 1.2

1 (a) $\frac{20}{7}$ (b) $\frac{5}{4}$ (c) $\frac{14}{9}$ (d) $\frac{4}{5}$

(e) $\frac{2}{11}$ (f) $\frac{8}{33}$ (g) $\frac{2}{7}$ (h) $\frac{6}{35}$

(i) $\frac{3x^2}{y}$ (j) $\frac{3x}{y}$ (k) $\frac{5x^2}{4y}$ (l) $\frac{5x^2}{6y^2}$

(m) $\frac{15x^4}{2y}$ (n) $\frac{3y^4}{2xz}$ (o) $\frac{3xy^2}{5z}$ (p) $\frac{5a^2}{12x^3yz^2}$

2 (a) $x=\sqrt{\frac{A}{2\pi}}$ (b) $x=\sqrt[3]{\frac{3V}{4\pi}}$ (c) $x=\frac{u+v}{2t}$ (d) $x=\sqrt{\frac{3W}{2\pi h}}$

3 (a) $\frac{1+x}{1+3x}$ (b) $\frac{2+x}{3-x}$ (c) $\frac{3+2x}{2+x}$

4 (a) $\frac{3x+5}{(x-1)(x+3)}$ (b) $\frac{x+7}{(x-3)(x+2)}$ (c) $\frac{x+3}{(2x-1)(3x+2)}$

(d) $\frac{x+5}{x+2}$ (e) $\frac{2x-3}{x-1}$ (f) $-\frac{x+3}{x+1}$

(g) $\dfrac{6x-1}{4(2x-1)(2x+1)}$

5 (a) $\dfrac{2x+1}{\sqrt{x}}$ (b) $\dfrac{3x+3}{\sqrt{x+3}}$ (c) $\dfrac{2x-2}{\sqrt[3]{x-2}}$

6 (a) $1+\dfrac{6}{x-5}$ (b) $1+\dfrac{2}{x+1}$ (c) $1-\dfrac{3}{x+5}$ (d) $1-\dfrac{4}{x-2}$

7 (a) $1 = A(x+1) + B(x-2)$

(b) $x + 2 = A(x-3) + B(x+2)$

(c) $2 = A(x+2)(x-3) + B(x+1)(x-3) + C(x+1)(x+2)$

To clear fractions in part (d) you multiply both sides by $(x-2)^2(x+1)$, *NOT* by $(x-2)^2(x-2)(x+1)$.

(d) $1 = A(x-2)(x+1) + B(x+1) + C(x-2)^2$

(e) $1 = Ax(x+2) + B(x+2) + Cx^2$ [again, NOT x^3 anywhere]

Exercise 1.4

1 (a) $12x$ (b) $4x^2$

(c) $2 + 3x$ (d) $12x$ (e) $3 + y$

(f) $3xy$ (g) $\dfrac{2x+5y}{4x+3y}$ (h) $\dfrac{x-2y}{3x-y}$

(i) can't be simplified (j) $\dfrac{2}{3}$ (k) $\dfrac{y}{2}$

(l) can't be simplified (m) -1 (n) -1

2 (a) $x = \dfrac{bpy}{aqz}$ (b) $x = \dfrac{4by^2}{3\pi aqz}$

3 (a) $\dfrac{6x}{abr^3}$ (b) $\dfrac{3h}{2br^3}$

4 [see Example 5]

(a) $(x-3)^2(5x-14)$ (b) $(2x+1)^3(14x+5)$

(c) $\frac{1}{2}(k+1)(k+2)$ (d) $\frac{1}{6}(k+1)(k+2)(2k+3)$

5 (a) $\dfrac{x+4}{x-3}$ (b) $\dfrac{3x+4}{x+2}$

(c) $\dfrac{x+1}{x-1}$ (d) $\dfrac{x(2x-1)}{x-1}$

(e) $\dfrac{-1}{x^2\sqrt{x^2+1}}$ (f) $\dfrac{x-2}{2x^2\sqrt{1-x}}$

(g) $\dfrac{-1}{2x\sqrt{x}\sqrt{x+1}}$ (h) $\dfrac{3+2x}{3(1+x)}$

Exercise 1.5

The answer to the question “why not?” in example 1 (page 16) is that $x^2 + y^2$ has no simple square root. In particular it is not $x + y$. [Remember that $(x + y)^2 = x^2 + 2xy + y^2$.]

1 (2, 4), (3, 1) **2** (6, –3), (7, 0)

3 (1, –1), (–1, 0) **4** (1, 2), (–1, –2)

5 (2, –1), $\left(-\frac{38}{25}, \frac{41}{25}\right)$ **6** (7, 4), $\left(8, \frac{7}{2}\right)$

7 (–5, 4), $\left(\frac{19}{5}, -\frac{13}{5}\right)$ **8** (–1, 1), $\left(\frac{5}{3}, -\frac{7}{9}\right)$

9 (2, 1), $\left(\frac{1}{2}, 4\right)$ **10** (3, 6), $\left(\frac{9}{2}, \frac{9}{2}\right)$

11 (2, 1), $\left(-\frac{5}{9}, -\frac{14}{9}\right)$ **12** (–1, –2), $\left(\frac{38}{13}, -\frac{9}{13}\right)$

13 $\left(\frac{5}{3}, \frac{1}{3}\right)$, $\left(-\frac{3}{5}, -\frac{4}{5}\right)$ **14** (2, –2) (only)

15 (6, –5) (only) **16** (6, 1), $\left(\frac{14}{3}, \frac{7}{3}\right)$